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On the Possibility of Superluminal Neutrino Propagation

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Abstract

We analyze the possibility of superluminal neutrino propagation $\delta v \equiv (v - c)/c > 0$ as indicated by OPERA data, in view of previous phenomenological constraints from supernova SN1987a and gravitational Čerenkov radiation. We argue that the SN1987a data rule out $\delta v \sim (E_\nu/M_N)^N$ for $N \leq 2$ and exclude, in particular, a Lorentz-invariant interpretation in terms of a ‘conventional’ tachyonic neutrino. We present two toy Lorentz-violating theoretical models, one a Lifshitz-type fermion model with superluminality depending quadratically on energy, and the other a Lorentz-violating modification of a massless Abelian gauge theory with axial-vector couplings to fermions. In the presence of an appropriate background field, fermions may propagate superluminally or subluminally, depending inversely on energy, and on direction. Reconciling OPERA with SN1987a would require this background field to depend on location.

1 Introduction

Data from the OPERA experiment have recently been interpreted [1] as evidence for superluminal ν_μ propagation between CERN and the Gran Sasso laboratory, with $\delta v \equiv (v - c)/c \sim 2.5 \times 10^{-5}$ for $\langle E_\nu \rangle \sim 28$ GeV¹. Such an extraordinary claim clearly requires extraordinary standards of proof, notably including confirmation by an independent experiment such as MINOS, T2K or NO ν A. Nevertheless, even while the OPERA data are undergoing experimental scrutiny, notably of the technical issues of pulse modelling, timing and distance measurement on which we are not qualified to comment, it may be helpful to present some relevant phenomenological and theoretical observations about the

¹OPERA used a similar experimental approach to that proposed in [2].

claimed effect. Here we report two sets of considerations concerning: (1) comparison with other phenomenological constraints on possible superluminal neutrino propagation, and (2) instructive theoretical toy models of Lorentz violation that exemplify the price to be paid to obtain such an effect. These toy models cast light on possible experimental probes of the OPERA effect.

As we show, reconciling this effect with other bounds on the propagation speeds of neutrinos, notably those provided by the supernova SN1987a [3, 4, 2], is a non-trivial issue. For example, if δv were independent of energy, the SN1987a neutrinos would have arrived at Earth years before their optical counterparts. This prematurity would have been even more pronounced for ‘conventional’ Lorentz-invariant tachyons, for which δv would *increase* at lower energies, forcing one to consider Lorentz-violating models. However, simple Lorentz-violating power-law modifications of the neutrino propagation speed $\delta v \sim (E_\nu/M_N)^N$ are also severely constrained by SN1987a. Specifically, constraints for $N = 1, 2$, derived previously in the paper [2] of which one of us (J.E.) was an author, are incompatible with the OPERA result for $\langle E_\nu \rangle \sim 28$ GeV [1]. Moreover, OPERA reports [1] that there is no significant difference between the values of δv measured for the lower- and higher-energy data with $\langle E_\nu \rangle \sim 13$ and 43 GeV, respectively, providing no indication that $N \neq 0$.

We also discuss the constraints imposed on superluminal neutrino propagation by Čerenkov radiation *in vacuo*. Electromagnetic Čerenkov radiation is suppressed by the absence of an electric charge for the neutrino [5]. However, *gravitational* Čerenkov radiation [6] is potentially significant for high-energy neutrinos, and an effect of the type reported by OPERA could suppress high-energy astrophysical neutrino signals such as those associated with the GZK cutoff and with gamma-ray bursters.

As a complement to these phenomenological remarks, we present two models for Lorentz-violating fermion propagation, with different energy dependences for the superluminality. One is a simple renormalizable Lifshitz-type fermion model in which the superluminality *increases quadratically* with energy. This model also exhibits dynamical fermion mass generation and asymptotic freedom. The other is a modification of an earlier gauge model for Lorentz violation with *subluminal* neutrino propagation that proposed previously by two of us (J.A. and N.E.M.) [7]. The modified model has a background axial U(1) gauge field, and may exhibit *superluminal* neutrino propagation with δv *falling as the inverse of the energy* and depending on direction. Since superluminal propagation with $\delta v \sim 2.5 \times 10^{-5}$ or greater is incompatible with the SN1987a data, such a model could be compatible with the data only if the background field depends on spatial location, and is enhanced in the neighbourhood of the Earth compared to its mean value along the line of sight to the Larger Magellanic Cloud.

2 Phenomenological Constraints on Superluminal Neutrino Propagation

The primary OPERA result on the mean neutrino propagation speed is

$$\delta v = (2.48 \pm 0.28 \pm 0.30) \times 10^{-5} \quad (1)$$

for ν_μ with $\langle E_\nu \rangle = 28$ GeV, where the errors in (1) are statistical and systematic, respectively. The OPERA Collaboration also provides the following supplementary information on the difference in mean arrival times of samples of higher- and lower-energy neutrinos with $\langle E_\nu \rangle = 13$ and 43 GeV, $\Delta t = 14.0 \pm 26.2$ ns, which corresponds to

$$\Delta(\delta v) = (0.57 \pm 1.07) \times 10^{-5} \quad (2)$$

for the difference $\Delta(\delta v)$ between the propagation speeds of these neutrino samples².

Constraints on possible deviations of the speed of ν_μ propagation from the velocity of light had been placed previously by the MINOS Collaboration [10], which found

$$\delta v = (5.1 \pm 2.9) \times 10^{-5} \quad (3)$$

for ν_μ with a spectrum peaking at $E_\nu \sim 3$ GeV and a tail extending above 100 GeV. The MINOS result (3) is not significant in itself, but is also compatible with the OPERA results (1, 2), as are earlier neutrino results [8].

However, more stringent constraints on models of neutrino propagation are imposed by the SN1987a neutrinos [3]. The observed neutrinos emitted by SN1987a had energies around three orders of magnitude smaller than the OPERA neutrinos. A significant fraction of them were undoubtedly ν_μ , and neutrino oscillation phenomenology severely constrains differences in the propagation speeds of different neutrino flavours, so the OPERA results (1, 2) may be confronted directly with the SN1987a data. Since the distance to SN1987a was ~ 50 kpc, i.e., $\sim 170,000$ light-years, an energy-dependent δv of the magnitude (1) would have caused the SN1987a neutrinos to have arrived over 4 years before their photon counterparts, whereas the maximum tolerable advance is only a few hours, corresponding to $\delta v \sim 2 \times 10^{-9}$ [4].

The SN1987a data are orders of magnitude more problematic for ‘conventional’ tachyonic neutrinos. Assuming Lorentz invariance, these would have a dispersion relation $E^2 = p^2 - \mu^2$, where $\mu^2 > 0$, and the corresponding deviation of the propagation speed from the velocity of light would be $\delta v \sim \mu^2/2E^2$. Thus, as the energy increases, the speeds of such ‘conventional’ Lorentz-invariant tachyonic neutrinos would *decrease* towards the velocity of light. Normalizing the effect to the value (1) of δv reported by OPERA for their relatively high-energy neutrinos would lead to an impossibly large effect for the SN1987a neutrinos. Moreover, the magnitude of μ^2 would be incompatible with limits on the ν_μ

²We note that no other experiment has made such accurate velocity measurements for any other particles with Lorentz boosts as large as the OPERA neutrinos.

mass from π and μ decay and, to the extent that oscillation experiments constrain the $\nu_e - \nu_\mu$ mass difference, also the direct constraint on the ν_e mass [9].

We are therefore led to consider the possibility of Lorentz violation. Although the OPERA Collaboration sees (2) no significant energy dependence of δv when comparing its lower- and higher-energy samples with $\langle E_\nu \rangle \sim 13$ and 43 GeV [1], the SN1987a data motivate us to look at the implications of an energy dependence $\delta v \sim (E_\nu/M_N)^N$ with $\delta v \sim 2.48 \times 10^{-5}$ for $\langle E_\nu \rangle \sim 28$ GeV. Under this hypothesis, the OPERA data would correspond to

$$M_1 \sim 1.1 \times 10^6 \text{ GeV}, \quad (4)$$

$$\text{or } M_2 \sim 5.6 \times 10^3 \text{ GeV}, \quad (5)$$

for linear and quadratic energy dependences, respectively. However, stringent constraints on M_1 and M_2 have been imposed previously by observations of the neutrino burst from SN1987a [3], which would have been spread out by any energy-dependence of δv . The following constraints on superluminal neutrino propagation were established by a collaboration including one of the present authors (J.E.) [2]³:

$$M_1 \sim 2.5 \times 10^{10} \text{ GeV}, \quad (6)$$

$$\text{or } M_2 \sim 4 \times 10^4 \text{ GeV}. \quad (7)$$

We recall that the supernova neutrino burst is expected to have contained large fractions of ν_μ and $\bar{\nu}_\mu$, so that these constraints apply *a priori* to the ν_μ used by OPERA in their measurement.

Comparing (4) with (6), we infer that a linear dependence of δv is not compatible simultaneously with the OPERA and SN1987a data. The situation with a quadratic energy dependence, cf, (5) and (7), is not in such stark contradiction with the SN1987a data, but the latter would prefer a stronger energy dependence that would be even more difficult to reconcile with the lack of any indication of energy dependence within the OPERA data (2). As for a possible constant δv , we recall that the OPERA measurement of $\delta v \sim 2.5 \times 10^{-5}$ would have led to the SN1987a neutrino signal being observed ~ 4 years before the optical signal, whereas the observed advance of < 3 hours (which is compatible with models of supernova explosions) would correspond to $\delta v < 2 \times 10^{-9}$ [4]. We infer that only an energy dependence of $\delta v \sim E^N$ with $N > 2$ could reconcile the OPERA and SN1987a data, though this is unlikely to be compatible with the lack of a significant energy dependence observed within the OPERA energy range (2).

The possibility of gravitational Čerenkov radiation has been studied in [6]. The case studied there was that of a particle propagating at the speed of light emitting *subluminal* gravitational radiation, but the same analysis applies to a *superluminal* particle emitting gravitational radiation travelling at the speed of light. It was shown in [6] that a particle

³The prospective sensitivity of the OPERA experiment to possible superluminal neutrino propagation was also estimated in [2], using a similar technique and with results similar to those now obtained (4, 5) by OPERA.

would lose all its energy within a time

$$t_{max} = \frac{M_P^2}{(n-1)^2 E^3}, \quad (8)$$

where $M_P \sim 1.2 \times 10^{19}$ GeV is the Planck mass and n is the refractive index: $n = 1 - \delta v$ in our case. Setting $\delta v \sim 2.5 \times 10^{-5}$ as suggested by OPERA, using (8) we find

$$t_{max} \sim \frac{2 \times 10^8}{[E_\nu(\text{GeV})]^3} \text{ years}. \quad (9)$$

We conclude that applying the OPERA result simple-mindedly would exclude by many orders of magnitude the observation of GZK neutrinos [11], which should have $E_\nu \sim 10^{10}$ GeV and propagate $\sim 10^8$ light-years⁴. Alternatively, neutrinos with $E_\nu \sim 2 \times 10^6$ GeV, the minimum for which the IceCube experiment has so far published an upper limit on the flux [12], could not travel more than $\sim 10^{-4}$ seconds, ample to explain their non-observation, though this surely has a less radical explanation! Conversely, observation of neutrinos violating the bound (9) would invalidate the hypothesis of a constant δv with the magnitude suggested by OPERA.

3 Lorentz-Violating Models with Superluminal Fermion Propagation

In light of the foregoing phenomenological discussion, one might be tempted to lose interest in theories with superluminal neutrino propagation. However, the effect reported by OPERA is so striking and of such potential significance that it is important to study whether such an effect is possible, even in principle, and how theoretical possibilities could be constrained by future experiments. In this Section, we show how to construct examples within the general framework of field theories with higher-order spatial derivatives, and discuss some characteristic experimental signatures.

3.1 Lifshitz-type Field Theory

Such theories have recently attracted renewed attention because of their improved convergence properties (for a recent review see [13] and references therein). In this spirit, a renormalizable Lifshitz-type theory of gravity has been proposed, which could lead to a renormalizable quantum gravity theory at high energies [14]. Such theories are free of ghosts, since the order of the time derivative in the action remains minimal, so that no new poles appear in particle propagators. However, in general such theories violate Lorentz symmetry at high energies [15].

⁴As for possible GeV-range neutrinos emitted by gamma-ray bursters with cosmological redshifts, (9) shows that they would lose their energy before reaching the Earth, in addition to arriving at very different times from their optical counterparts if δv is given by (1).

We now exhibit a Lifshitz-type model with superluminal fermion propagation. The model is formulated in three space dimensions with anisotropic scaling parameter $z = 3$. In this scenario, the mass dimensions of the coordinates are $[t] = -z = -3$, $[x] = -1$, and the free fermion action is [16]

$$S_{4ferm} = \int dt d\vec{x} \left(\bar{\psi} i \gamma_0 \dot{\psi} - \bar{\psi} (M^2 - \Delta) (i \vec{\partial} \cdot \vec{\gamma}) \psi + g (\bar{\psi} \psi)^2 \right), \quad (10)$$

where $\Delta \equiv -\partial_i \partial^i = \vec{\partial} \cdot \vec{\partial}$, and we use $(+1, -1, -1, -1)$ as the metric signature. The model is renormalizable since $[g] = 0$, and we have also $[M] = 1$, $[\psi] = 3/2$. This model exhibits asymptotic freedom as well as the dynamical generation of a mass m_{dyn} for the fermion, as discussed in more detail in [16], [13].

Taking this fermion dynamical mass into account, we obtain the following dispersion relation:

$$\omega^2 = m_{dyn}^2 + M^4 p^2 + 2M^2 p^4 + p^6, \quad (11)$$

and, assuming that $M \neq 0$, it is possible to recover approximately Lorentz-invariant kinematics in the infra-red limit, since the rescaling $\omega = M^2 \tilde{\omega}$ leads to

$$\tilde{\omega}^2 = \mu_{dyn}^2 + p^2 + \frac{2}{M^2} p^4 + \frac{p^6}{M^4}, \quad (12)$$

where $\mu_{dyn} \equiv m_{dyn}^3/M^2$. Using this dispersion relation, one can compute the group velocity $\frac{\partial \tilde{\omega}}{\partial p}$ and the phase velocity $\frac{\tilde{\omega}}{p}$ as power series in $(p/M)^{2N} p$: $N \geq 1$. The superluminal character of both follows immediately from (12), with the first correction of order p^2/M^2 , which is quadratically suppressed by the Lorentz-violating mass scale M . We note that the *superluminality* of this model is an unavoidable consequence of the relative signs of the various terms appearing in (10), if one is to avoid tachyonic modes for sufficiently high momenta p .

It is clear from (11) that the superluminality δv *increases* quadratically with the fermion momentum (or energy) for $p(E) < M$, and even faster at higher momenta (energies). As discussed in the previous Section, such a quadratic dependence is not easy to reconcile with the lack of energy dependence in δv seen in OPERA data [1], though it comes closer to compatibility between the OPERA data and the constraint imposed by SN1987a [2].

The fact that the superluminality in this model is quadratic in E_ν implies that *no effect* should be seen at the level (1) in the MINOS and T2K experiments, since they have mean energies that are almost an order of magnitude lower than the CNGS beam. In particular, the indication that $\delta v \neq 0$ from previous MINOS data [10] would not be confirmed in this scenario.

3.2 Lorentz-Violating Gauge Theory

We now consider more complicated models that lead to forms for δv with very different energy dependences, involving a fermion coupling to either a vector or an axial U(1) gauge field. If there is a background field with a suitable constant value in a given reference frame,

these models may exhibit superluminal fermions, as well as other dramatic signatures highlighted below. These models are concrete realizations of the ideas of [17], where the phenomenology of Lorentz violation has been discussed in models where the maximal speeds for various particles depend on the species.

A minimal Lorentz-violating (LV) extension of massless Quantum Electrodynamics (QED) was proposed in [18], in which higher-order spatial derivatives were introduced for the photon field, and fermions remained minimally coupled to the photon. This theory has the features that the light-cone ‘seen’ by fermions differs from that ‘seen’ by the photon. Specifically, in the theory of [18] (i) the photon always travels at the conventional speed of light, (ii) fermions travel *subluminally*, and (iii) fermion masses may be generated dynamically in such a framework, as an alternative to the Higgs mechanism. We will show that similar theories with a background vector or axial U(1) field (see also [7]) may lead to *superluminal* fermion propagation, albeit with no mechanism for fermion mass generation.

The Lagrangians of the models read:

$$\mathcal{L}_{V,A} = -\frac{1}{4}G_{\mu\nu} \left(1 - \frac{\Delta}{M^2}\right) G^{\mu\nu} + \bar{\psi} (i \not{\partial} - g_{V,A} \not{\Gamma} \Gamma_{\mathcal{I}}) \psi - m \bar{\psi} \psi, \quad (13)$$

where $G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ and B_μ is a gauge field with either a vector coupling g_V or an axial coupling g_A , depending whether $\Gamma = 1$ or γ_5 , respectively. The presence of an axial $\gamma_5 \gamma^\mu$ fermion/gauge boson vertex would introduce the possibility of chiral anomalies, which could be cancelled by suitable choices of the couplings to the different fermion fields $\psi = (\psi_1, \dots, \psi_n)$, represented here by the matrix \mathcal{I} with the property:

$$\text{tr}\{\mathcal{I}\} = 0. \quad (14)$$

In the case of a doublet of fermions, for definiteness, one could use

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \mathcal{I} \equiv \frac{1}{\sqrt{2}} \tau_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \text{tr}(\mathcal{I}^2) = 1, \quad (15)$$

although other choices can be made [19], as long as the anomaly-free condition (14) is satisfied. It should be noted that *no* higher-order spatial derivatives are introduced for the fermion fields because, in order to respect gauge invariance, such terms would need to be of the form

$$\frac{1}{M^{n-1}} \bar{\psi} (i \vec{D}_5 \cdot \vec{\gamma})^n \psi \quad n \geq 2, \quad (16)$$

whereby D_5 denotes the axial-gauge-field fermion covariant derivative, which would introduce new, non-renormalizable couplings. The Lorentz-violating modification proposed in the Lagrangian (13) does *not* alter the photon dispersion relation, which remains relativistic, but *does* modify the fermion propagator, as we discuss below.

It was observed in [20] that models of this type can be obtained by considering the propagation of photons and charged fermions in a D-particle model of space-time foam [21], according to which our world is viewed as a 3-brane propagating in a higher-dimensional bulk space that is punctured by point-like D0-brane defects (D-particles). Such models may

lead to non-trivial optical properties of the vacuum, because electrically-neutral matter excitations, such as photons and neutrinos, may acquire non-trivial refractive indices through non-trivial interactions with the D-foam. In previous D-foam models [21], these interactions led to *subluminal* propagation. In the flat space-time limit, where the low-energy Lagrangian is derived, the microscopic reason why fermions do not have higher-derivative modifications was that charge conservation forbids interactions of charged fermions with the foam.

The explicit Lorentz violation due to the higher-spatial-derivative term in the action (13) implies that the light-cone ‘seen’ by the fermions is different from that ‘seen’ by the gauge boson, assumed here to be (almost) massless like the photon, always travels at c , the speed of light *in vacuo*. Specifically, the maximal speed for the fermions is *smaller* than c , as in the vector models of refs. [18, 7]. This may be seen by following the one-loop analysis of the fermion wave-function renormalization calculated in [18]. Due to the higher-order spatial derivatives in (13), the one-loop quantum corrections to the fermion kinetic terms are different for time and space derivatives. A similar computation as the one made in [18] yields corrections of the form

$$i\bar{\psi} \left((1 + Z_0) \partial_0 \gamma^0 - (1 + Z_1) \vec{\partial} \cdot \vec{\gamma} \right) \psi , \quad (17)$$

where

$$\begin{aligned} Z_0 &= -\frac{2\alpha_{V,A}}{\pi} \left(\frac{1}{4} \ln(1/\mu) + \ln 2 - \frac{1}{2} \right) + \mathcal{O}(\mu^2 \ln(1/\mu)), \\ Z_1 &= -\frac{2\alpha_{V,A}}{\pi} \left(\frac{1}{4} \ln(1/\mu) + \frac{25}{18} - \frac{5}{3} \ln 2 \right) + \mathcal{O}(\mu^2 \ln(1/\mu)) . \end{aligned} \quad (18)$$

where $\alpha_{V,A} \equiv g_{V,A}^2/4\pi$ and $\mu \equiv m/M$. We note that the dominant terms in (18), which are proportional to $\ln(1/\mu)$, are the same for Z_0 and Z_1 . This is to be expected since, in the Lorentz-invariant limit: $M \rightarrow \infty$ and hence $\mu \rightarrow 0$ for fixed fermion mass, we must have $Z_0 = Z_1$. After redefinition of the bare parameters in the minimal subtraction scheme, where only the terms proportional to $\ln(1/\mu)$ are absorbed, the fermion dispersion relation is

$$\left(1 - \frac{\alpha_{V,A}}{\pi} [2 \ln 2 - 1] \right)^2 \omega^2 = \left(1 - \frac{\alpha_{V,A}}{\pi} [25/9 - (10/3) \ln 2] \right)^2 p^2 + m^2 . \quad (19)$$

and the fermion phase and group velocities v_ϕ , v_g are both subluminal:

$$v_\phi = v_g = 1 - \frac{\alpha_{V,A}}{\pi} \left(\frac{34}{9} - \frac{16}{3} \ln 2 \right) + \mathcal{O}(\alpha_{V,A}^2) < 1 , \quad (20)$$

whereas the gauge boson B_μ propagates with the standard speed of light *in vacuo*, c , as required by gauge invariance [18].

A few remarks are in order at this point. First: in view of (20), the above models constitute explicit microscopic realizations of the class of Lorentz-violating theories of the type considered in [17], with species-dependent light cones. Secondly, the fact that the constant wave function renormalization (18) is found to be less than one, which leads to the subluminal velocities (20), is a rather general property of quantum field theory, stemming from

unitarity [22]. Indeed, in field theories with non-negative-metric states, the wave function renormalization A must satisfy $0 < A < 1$, which also implies non-negative anomalous dimensions. However, there may be cases, e.g., with derivative interactions [22], in which negative anomalous dimensions appear, with the consequence that the wavefunction renormalization can be larger than one. It would be interesting to investigate the possibility of superluminal propagation in such cases, by analogy with the scenario discussed above.

We now explore the possibility of superluminal fermion propagation in the context of the above theories. To this end, we first consider the possibility of a constant background gauge field $B_\mu^{(0)}$, in which case the relevant part of the action (13) reduces to:

$$\mathcal{L}_{\text{bckgrd}} = \bar{\psi} \left(i \not{\partial} - g_{V,A} \not{B}^{(0)} \Gamma_{\underline{I}} \right) \psi - m \bar{\psi} \psi . \quad (21)$$

These models fall within the general category of Lorentz-violating extensions of the Standard Model [15], as reviewed in the specific case of neutrinos in [23], taking into account the available neutrino oscillation data. Depending on the sign of the background field $B^{(0)}$ in (21), one may have group velocities for the fermions which are superluminal. The quantum fluctuations of the axial gauge field would tend to counteract such superluminality, as discussed above (20). Nevertheless, for sufficiently weak couplings $\alpha_{V,A}$ and a sufficiently strong background field $B^{(0)}$, the maximal fermion speed may be superluminal. A value of $\delta v \sim 2.5 \times 10^{-5}$, as reported by OPERA [1], may be arranged in these models with a small, perturbative gauge coupling $g_{V,A} < 1$ and a background field $B^{(0)}$ of appropriate magnitude.

The vector (axial) interaction of (21) has the same (different) signs for left- and right-handed fermions, such as neutrinos and their antiparticles ψ^c , which we assume to be Majorana fermions. This could lead to a physically important difference between the dispersion relations of neutrinos and antineutrinos, and hence apparent CPT violation:

$$\begin{aligned} \omega_\nu &= \sqrt{(\vec{p} - g_{V,A} \vec{B})^2 + m^2} + g_{V,A} B_0 , \\ \omega_{\bar{\nu}} &= \sqrt{(\vec{p} \mp g_{V,A} \vec{B})^2 + m^2} \pm g_{V,A} B_0 . \end{aligned} \quad (22)$$

where the upper (lower) symbols in the combinations \pm, \mp refer to the vector (axial) case ^{5,6}. Notice that these dispersion relations are the usual ones for massive particles, though with generalised momenta

$$\Pi^0 = \omega_\nu \mp g_{A,V} B^0, \quad \vec{\Pi} = \vec{p} \mp g_{A,V} \vec{B}, \quad (23)$$

where the upper signs apply to neutrinos, and to antineutrinos with a vector interaction, and the lower signs apply to antineutrinos with an axial interaction.

⁵In the simple two-flavour axial model (15), the particle of one flavour would exhibit the same dispersion relation as the antiparticle of the other flavour.

⁶An effect similar to the axial case in (22), but without the flavour structure, could arise purely geometrically in the propagation of fermions in space-times that break rotational symmetry, such as rotating Kerr black holes or axisymmetric Robertson-Walker Universes, as discussed in [24]. Such geometric effects stem from the coupling of the spin of the fermions to non-trivial local curvature effects that arise in such space-times.

Assuming that the components \vec{B}, B_0 are constants in a local frame of reference, and defining the angle between the three-vectors \vec{p} and \vec{B} to be ϑ , we may write the phase velocity following from (22) for high-energy neutrinos with $p \gg m$ as:

$$v_{ph} = \frac{\omega_\nu}{p} \simeq 1 \mp \frac{g_{V,A}}{p} (|\vec{B}| \cos \vartheta - B_0) + \dots, \quad (24)$$

where dots represent higher orders in $1/p$. We obtain a similar expression for antineutrinos but with the replacement $|\vec{B}| \rightarrow -|\vec{B}|$ and $B_0 \rightarrow -B_0$ in the axial case. However, the superluminality associated with (24) does not apply to the group velocity, which is subluminal:

$$v_g = \frac{\partial \omega_\nu}{\partial p} = 1 - \frac{1}{2p^2} (g_{V,A}^2 B^2 \sin^2 \vartheta + m^2) + \dots, \quad (25)$$

which is the same for neutrinos and antineutrinos, and where the dots represent higher orders in $1/p$. Note that $|v_g - 1|$ is of order $1/p^2$, unlike the case of the phase velocity, where $|v_{ph} - 1|$ is of order $1/p$ ⁷.

As a further step, we modify the background space-time in which the neutrino propagates, exhibiting an extension of this model with superluminal group velocities. We embed the model (21) in a modification of Minkowskian space-time with non-diagonal metric components that break the rotational symmetry along a specific axis,

$$g_{0i} = \vec{V}_i, \quad i = 1, 2, 3 \quad (26)$$

where $|\vec{V}| \equiv V \ll 1$ is considered as a small perturbation⁸. For *constant and homogeneous* \vec{V} , the dispersion relations (22) for neutrinos are modified to

$$\Pi^\mu \Pi^\nu g_{\mu\nu} = m^2, \quad (27)$$

where Π^μ is given by (23). From this we obtain:

$$\begin{aligned} \omega_\nu &= -(\vec{p} - g_{V,A} \vec{B}) \cdot \vec{V} + \sqrt{(\vec{p} - g_{V,A} \vec{B})^2 + m^2} + g_{V,A} B_0 + O(V^2), \\ \omega_{\bar{\nu}} &= -(\vec{p} - g_{V,A} \vec{B}) \cdot \vec{V} + \sqrt{(\vec{p} \mp g_{V,A} \vec{B})^2 + m^2} \pm g_{V,A} B_0 + O(V^2). \end{aligned} \quad (28)$$

Assuming that the components \vec{B}, B_0 are constants in a local frame of reference, considering for simplicity the case with $|\vec{V}| \ll |\vec{B}|$, and defining the angle between the three-vectors \vec{p} and \vec{B} to be ϑ , and that between \vec{p} and \vec{V} to be φ , then we observe that Eq. (28)

⁷This model has the interesting feature that the (anti)neutrino propagation velocity depends on the direction of propagation. This example raises the possibility that, if a constant limiting velocity of light does not apply to neutrinos, perhaps the Michelson-Morley experiment should also be revisited for neutrinos?

⁸This is motivated by the suggestion that the metric distortion (26) and the axial background case (21) may have a common geometric origin, given that they may both be associated with background space-time effects, with the vector \vec{B} pertaining to the coupling of the (anti)neutrino spin to local curvature effects [24], as mentioned above.

yields the following expressions for the neutrino phase and group velocities for relatively high-energies: $p = |\vec{p}| \gg m, |\vec{B}|$:

$$\begin{aligned} v_{ph} &= 1 - V \cos \varphi + \frac{g_{V,A} \vec{B} \cdot \vec{V}}{p} - \frac{g_{V,A}}{p} (|\vec{B}| \cos \vartheta - B_0) + \dots \\ v_g &= 1 - V \cos \varphi - \frac{g_{V,A}^2 B^2 \sin^2 \vartheta + m^2}{2p^2} + O(V^2), \end{aligned} \quad (29)$$

and similarly for antineutrinos but with the replacement $B \rightarrow -B$ in the axial case. Superluminal group velocities of order $\delta v \sim 2.5 \times 10^{-5}$, as reported by the OPERA experiment [1], could be obtained for suitable values of the combination $-V \cos \varphi > 0$.

The model (21, 26, 29) has several dramatic and testable consequences:

- The deviation of the neutrino propagation speed from that of light could exhibit non-trivial dependence on E_ν , due to the combination of terms in (29), that is not a simple power law. Thus, compatibility with the MINOS result [10] is a non-trivial issue, which we address below.

- The neutrino group velocity would depend on the angle of propagation. This means that the speed of propagation would, in general, vary sinusoidally during the sidereal day, and could even vary between super- and subluminality⁹. This modulation would be absent only for \vec{V} oriented parallel to the Earth's rotational axis.

- The amount of superluminality would also, in general, depend on the geographical orientation of the neutrino beam. For example, in the hypothetical example in which \vec{V} is oriented parallel to the Earth's rotational axis, the sign of the effect on neutrinos travelling northwards (cf, the Fermilab-Soudan neutrino beam) would be opposite to beams travelling in a southerly direction (cf, the CNGS neutrino beam), and would be almost null for a beam oriented almost east-west (cf, the T2K neutrino beam). Studying the compatibility of MINOS data [10] with this model must therefore take into account the ambiguity in the orientation of \vec{V} , as well as the energy dependence of the superluminal effect in this model.

- It is possible that the orientation and magnitude of \vec{V} and \vec{B} vary on an interstellar scale, in which case the SN1987a constraint on the neutrino velocity applies only to an average over space and time of the possible superluminality effect, and there is no *a priori* contradiction with the OPERA result.

- If the neutrino group velocity is superluminal, the corresponding *antineutrino* group velocity in the same direction would also be *superluminal* in both the axial and vector cases.

Another possibility is that the vector \vec{V} (26) may be associated with distortions of space-time due to the interaction of the neutrino with space-time defects, as in stringy D-particle models of space-time foam [21], in which the vector \vec{V} is associated with the average transfer of momentum from the neutrino to space-time defects with which it interacts during its propagation. In such a case, the metric would be of Finsler type, i.e., depending not only on the space-time coordinates but also on momenta. This possibility

⁹However, we would not expect any day-night or seasonal dependence, which is consistent with the absences of such effects in the OPERA data [1].

is included within our formalism, but we do not pursue it further here. We note, however, that in such models electric charge conservation (which is enforced by gauge invariance) prevents charged matter (such as electrons) from interacting non-trivially with the D-particle foam [21, 20], so that only neutral excitations (such as photons and neutrinos) may be affected by the foam. This may provide a microscopic explanation of the fact that for electrons no deviations from special relativity have been observed with a precision $\sim 10^{-9}$ [5].

4 Summary and Prospects

The report from OPERA of superluminal neutrino propagation is very surprising, and it may well not survive further scrutiny. Moreover, as we have shown in the earlier part of this paper, it is subject to constraints from studies of lower-energy neutrinos, specifically those emitted by SN1987a [2], and would have implications for higher-energy astrophysical neutrinos. In particular, we have argued that the SN1987a data exclude a ‘conventional’ Lorentz-invariant tachyonic neutrino interpretation of the OPERA data. On the other hand, as we have shown through the toy models presented in the latter part of this paper, it is possible to construct Lorentz-violating theories in which neutrinos travel faster than photons, which always travel at c . We have exhibited such models in which the superluminality either increases or decreases with energy. Superluminal neutrinos should not be discarded as a phenomenological impossibility, but rather regarded as a scenario to be probed and constrained by experiment. In particular, we have shown that the effect could depend on the orientation of the neutrino beam. For the moment, the OPERA measurement provides a stimulus for investigating such scenarios, but Lorentz-violating superluminal fermion propagation should not necessarily be discarded out of hand, even if the OPERA result were not to be confirmed.

Notes added

A number of papers reacting to the OPERA effect [1] appeared before ours [25]. There is some overlap with the phenomenological considerations presented in [2] and here, but the models discussed here do not seem to have been discussed yet in this context.

We also note that, among the extensive literature since our paper was released, it has been pointed out [26] that the modified Čerenkov radiation process $\nu \rightarrow \nu e^+ e^-$ is potentially an important mechanism for energy loss by superluminal neutrinos. A first direct experimental limit on this process and on the distortion of the neutrino energy spectrum that it might induce has been reported [27]. We limit ourselves here to noting that the rate for this process is very sensitive to the magnitude of δv , and also to its energy dependence. We leave for future work a detailed combined study of the interplay between this and other constraints, pending verification of the magnitude of the OPERA result and its energy dependence. .

In this context, we also note that in any model with general coordinate invariance, such as our model (21), where the modified dispersion relations (27) arise as a result of a non-trivial metric background, e.g., (26), one may always find coordinate transformations to a frame in which the superluminal effects are absent. For the background (26), responsible for the superluminal V -dependent parts of the group velocity (29), such transformations are of the Galilean form $t \rightarrow t$, $x^i \rightarrow x^i - V^i t$, which, from the point of view of a passive observer, result in a change in the metric $\delta g_{0i} = -V_i$ that can cancel the superluminal effects in the dispersion relation in that frame. Since the Čerenkov radiation is a physical (observer-independent) phenomenon, it cannot depend on the coordinate choice made by the observer, whereas the refractive index can, being frame-dependent. Hence, we conclude that the arguments of ref. [26] do not apply directly to our second model. We note that this argument would imply that, in the transformed frame, the dispersion relations of other particles, such as electrons, are affected. However, this is not in contradiction with the current bounds for these particles, which are derived in different experimental conditions, and specifically in a different reference frame.

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